Estimating exposure from Census Populations

(updated 30 Apr 18 in response to a correction from Iván Williams. Thanks, Iván.)

We want to estimate exposure over three calendar years (census year-1, census year, census year+1) from a census population.

Define

A = the lower bound of an age group (e.g., A=80)

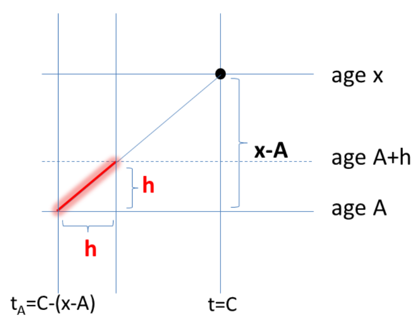
h = the width of an age group (e.g., h=1)

t = time measured from the beginning of (census year-1) (e.g. if we're using 2010 census data then t=0 corresponds to 1 Jan 2009, and t=3 to 1 Jan 2012).

C = the date of the census, measured on the t-scale (e.g., the 2010 Census was on 1 Aug 2010, so C=1.58)

x = the exact age of an individual on the census date (e.g., x=79.3)

Someone who was exactly x years old on the census date C was age x-(C-t) at time t, would have had their A^{th} birthday at time $t_A(x)=C-(x-A)$, and would be in the [A,A+h) age group between times $t_A(x)$ and $t_A(x)+h$.



Given any reasonable approximation to the life table survival function p(x) we can estimate the number of person-years lived in [A,A+h) over times [0,3), per person x years old on the census date, as

$$PY_A(x) = \frac{1}{p(x)} \int_0^3 I(t \geq C-(x-A)) \cdot I(t < C-(x-A)+h) \cdot p(x-(C-t))\ dt$$

where I( ) is a (0,1) indicator function for the expression in parentheses. We approximate the integral as a sum over a grid of 60 times t_i= .025, .075,..., 2.975 as

$$PY_A(x) \approx \frac{1}{20} \frac{1}{p(x)} \sum_i I(t_i \geq C-(x-A)) \cdot I(t_i < C-(x-A)+h) \cdot p(x-(C-t_i))$$

The last step in the approximation is to assume that those counted at integer age X in the census are uniformly distributed over 5 exact ages x=X+.10,X+.30,...,X+.90, so that per integer-X-year-old recorded in the census there were

$$PY_{AX} \approx \frac{1}{5} \left[\; PY_A(X+.1) + PY_A(X+.3) + \cdots + \; PY_A(X+.9) \; \right]$$

person-years of exposure at ages [A,A+h) over times [0,3).

An example

Using the same average male mortality rates in the HMD that we used for our TOPALS standard, each 55-year old male observed in the 2010 Census (on 1 Aug 2010, t=1.58) represent an average of

0.18 person-years of exposure at ages [53,54) over 2009-2011
0.93 person-years of exposure at ages [54,55) over 2009-2011
1.00 person-years of exposure at ages [55,56) over 2009-2011
0.81 person-years of exposure at ages [56,57) over 2009-2011
0.08 person-years of exposure at ages [57,58) over 2009-2011

Similarly, a 90-year-old male observed in the 2010 Census represents an average of

0.24 person-years of exposure at ages [88,89) over 2009-2011
1.14 person-years of exposure at ages [89,90) over 2009-2011
1.00 person-years of exposure at ages [90,91) over 2009-2011
0.66 person-years of exposure at ages [91,92) over 2009-2011
0.06 person-years of exposure at ages [92,93) over 2009-2011

The figure below illustrates exposure calculations for a male exactly 55.5 years old on the 2010 census date.